# ASYMPTOTIC ACCELERATED EXPANSION IN STRING THEORY

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with Irene Valenzuela and José Calderón Infante

String Pheno '22, July 7 2022

Instituto de Física Teórica UAM-CSIC

# Outlook of the talk

1. Accelerated expansion of the Universe
2. Where can we obtain our potentials from?
3. Eqs. of motion and gradient flow
4. Two detailed examples
5. General results
6. Conclusions + Outlook

# Accelerated expansion of the Universe

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What other options can we try in String Theory to realize an accelerated universe?

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What other options can we try in String Theory to realize an accelerated universe?

**QUINTESSENCE**

**(asymptotically)**

# Accelerated Universe from a running potential



Consider a FLRW Universe dominated by $\{\varphi^a\}_{a=1}^n$ scalar fields.

$$S = \int \mathrm{d}^d x \sqrt{-g} \left\{ \frac{R}{2} + \frac{1}{2} g^{\mu\nu} G_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b - V(\varphi) \right\}$$

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Under **slow-roll** approximation we will have accelerated expansion if

$$\epsilon = -\frac{H'}{H^2} = \frac{d-2}{4} \frac{\|\nabla V\|^2}{V^2} \left( 1 + \frac{\Omega^2}{(d-1)^2 H^2} \right)^{-1} < 1$$

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Equivalently,

$$\gamma=\frac{\|\nabla V\|}{V}<\frac{2}{\sqrt{d-2}}$$

**dS coefficient**

# Swampland de Sitter Conjecture

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Difficulties obtaining stable dS vacua motivate

$$\gamma = \frac{\|\nabla V\|}{V} > c_d$$

[Obied, Ooguri, Spodyneiko, Vafa, '18]

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$$\gamma = \frac{\|\nabla V\|}{V} > c_d$$

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Some **lower bounds** for **asymptotic** $\gamma$ have been proposed:

$$c_d^{\text{strong}} = \frac{2}{\sqrt{d-2}}$$

[Rudelius,'21]

$$c_d^{\text{TCC}} = \frac{2}{\sqrt{(d-1)(d-2)}}$$

[Bedroya,Vafa,'21]

# Where can we obtain our potentials from?

We consider scalar potentials coming from:

- Superpotential $W$

$$\left.\begin{aligned} K &= -\log \prod_{j=1}^{n} (s^j)^{d_j} \\ W &= \sum_{\vec{m}\in\mathcal{E}} \rho_{\vec{m}}(\phi) \prod_{j=1}^{\hat{n}} (i s^j)^{\frac{1}{2}(d_j+m_j)} \end{aligned}\right\}$$

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4d $\mathcal{N}$=1 F-term potential (Cremmer et al)

[Lanza,Marchesano, Valenzuela,'21]

$$V = e^K \left( K^{a\bar{b}} D_a W \overline{D_{\bar{b}} W} - 3|W|^2 \right)$$

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- Scalar potential from F-theory on $Y_4$ using **Asymptotic Hodge Theory**:

$$V_M = \frac{1}{\mathcal{V}_4^3}(\|G_4\|^2 - \langle G_4, G_4\rangle) \approx \frac{1}{\mathcal{V}_0^3}\left( \sum_{\vec{l}\in\mathcal{E}} \|\rho_{\vec{l}}(G_4,\phi)\|_\infty^2 \prod_{j=1}^{h^{3,1}(Y_4)} (s^j)^{\Delta l_j} - A_{\text{loc}} \right)$$

Allows for IIA and IIB settings!

[Grimm, Li, Valenzuela, '20]

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$$W = \sum$$

4d

$$K_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$$

**ONLY SAXIONS ENTER THE PICTURE!!!**

- otic Hodge Theory:

$$V_M \quad \mathcal{V}_0^3 \left( \sum_{\vec{l}\in\mathcal{E}} \|\rho_{\vec{l}}(G_4,\phi)\|_\infty^2 \prod_{j=1}^{h^{3,1}(Y_4)} (s^j)^{\Delta l_j} - A_{\text{loc}} \right)_{\text{All}}$$

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# Equations of motion and gradient flow

We need a potential such that $V \to 0$ along some trajectory towards infinity!

Scalars asymptotically follow gradient flow trajectories: Initial info is lost!

$$\dot{s}^j = -\mathcal{F}(\lambda)\,\partial^j V_M(\lambda)$$

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**Geodesics!!**

Only $\vec{\beta}$ and the metric $G_{ab}$ will enter in the dS coefficient $\gamma$

# How to obtain the gradient flow trajectories?

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Solving the gradient flow eqs. for $V$ with more than one term can be difficult!

Of all possible trajectories $\vec{\beta}$ we choose that for which $V$ decays faster:

$$V|_{\vec{\beta}}(\lambda) = \sum_{\vec{l}\in\mathcal{E}} \tilde{A}_{\vec{l}} \lambda^{\beta^i l_i} \Longrightarrow \boxed{\mathrm{P:} \min_{\hat{\beta}\in\mathbb{S}^{\hat{n}}} \left\{ \max_{\vec{l}\in\mathcal{E}} \left\{ \hat{\beta}^i l_i \right\} \right\}}$$

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Our solutions can be:

- Valleys: $\vec{l}^{\,\text{dom}} \in \mathcal{E}_{\text{rest}}$ (positive and negative $l_i^{\text{dom}}$)
- General trajectories: $\vec{l}^{\,\text{dom}} \in \mathcal{E}_{\text{light}}$ (only negative $l_i^{\text{dom}}$)

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Our solutions can be:

- Valleys: $\vec{l}^{\text{dom}} \in \mathcal{E}_{\text{rest}}$ (positive and negative $l_i^{\text{dom}}$) **Interesting stuff is found here!!**
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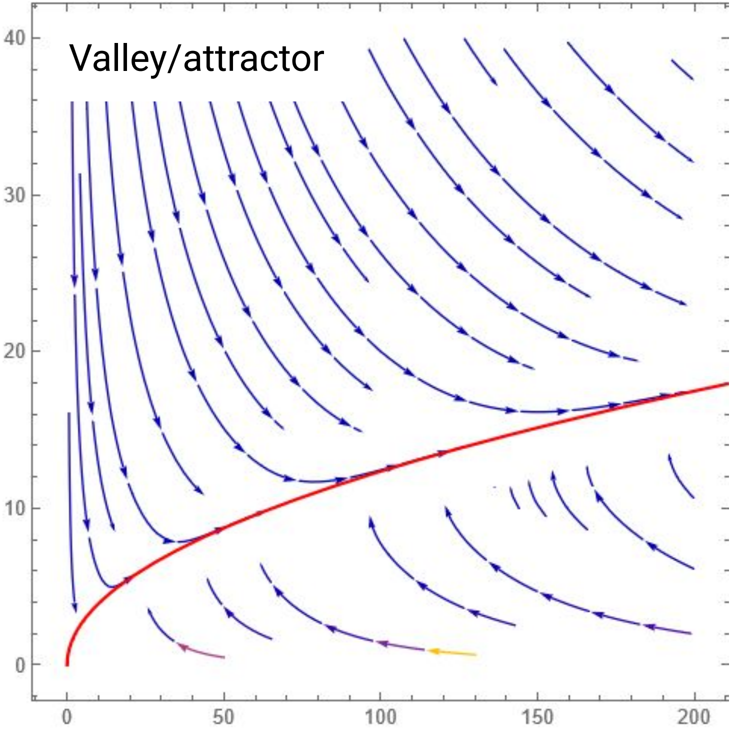
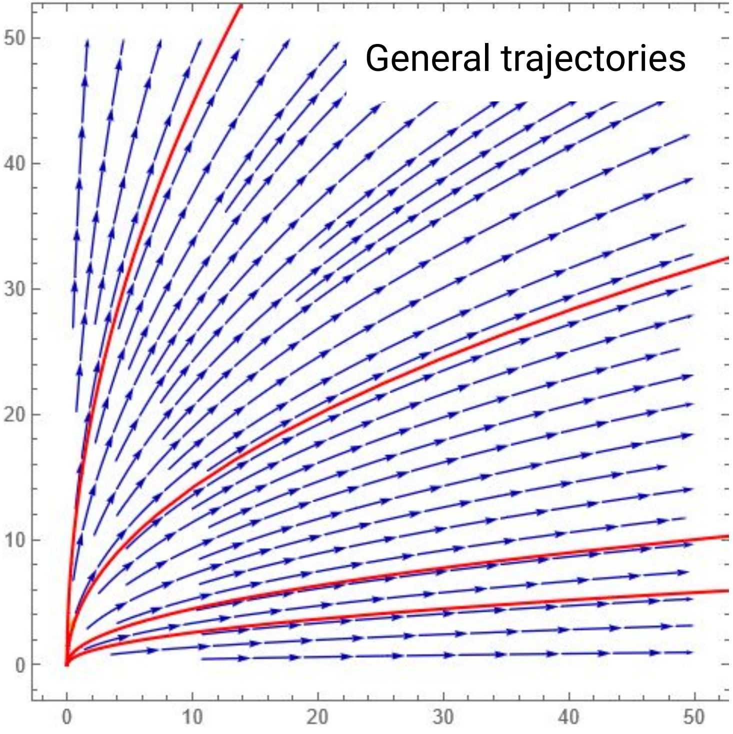
$$V = \frac{u}{s} + \frac{s}{u^3}$$




$$\vec{s}(\lambda) = \left(\sqrt{\frac{3}{7}}\lambda^2, \lambda\right)$$

$$V = \frac{1}{us^2}$$




$$\vec{s}(\lambda) = \left(\alpha_s \lambda^2, \lambda\right), \ \alpha_s > 0$$

# TWO DETAILED EXAMPLES!

# V from superpotential

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It is proposed in [Rudelius,'21] that **positive** scalar potentials coming from superpotential must fulfill $\gamma > 2\sqrt{\frac{d-1}{d-2}} > c_d^{\text{strong}}$ .

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This argument implicitly assumes we are working with **just one term** in $V$ and $W$

$$\left.\begin{aligned} W &= \rho_1(is) + \rho_2(is)^{-\frac{1}{2}}(iu)^{\frac{5}{2}} \\ K &= -\log(2s) - 3\log(2u) \end{aligned}\right\} V = \frac{\rho_1^2}{16}\frac{s}{u^3} + \frac{7\rho_2^2}{48}\frac{u^2}{s^2} \Longrightarrow \begin{cases} \vec{s}(\lambda) = \left(\left(\frac{49\rho_1^2}{18\rho_2^2}\right)^{\frac{1}{3}}\lambda^5, \lambda^3\right) \\ \boxed{\gamma = \sqrt{\frac{8}{13}}} < c_4^{\text{TCC}} < c_4^{\text{strong}} < \sqrt{6} \end{cases}$$

Now the normal components to the trajectory are not flat: They contribute to $\nabla V$ so that $\|\nabla(V_1 + V_2))\| < \|\nabla V_1\|, \|\nabla V_2\|$: **We can lower bounds!!**

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**PROBLEM**: Not obtained from F-th. +Asymptotic Hodge Theory 🤔

# $V_M$ from F/M-theory using Asymptotic Hodge Theory

Through orientifold compact. on a $h^{3,1}(Y_4) = 2$ CY 4-fold of F-theory, through singularity enhancement (II→V) we get type IIB ST at large volume limit :

$$V_M = \frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34}\frac{u}{s} + A_{52}\frac{s}{u^3}$$

[Grimm,Li,Valenzuela,'20]

depending on the CS moduli $(t^1, t^2) = (b + is, c + iu)$: Axions stabilized!

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This results in a $\vec{\beta} = (2, 1)$ trajectory with $\gamma = \sqrt{\frac{2}{7}}$ : VIOLATES $c_4^{\mathrm{TCC}}$ and $c_4^{\mathrm{strong}}$

# $V_M$ from F/M-theory using Asymptotic Hodge Theory



⚠️IMPORTANT: **No-scale structure** cancels $-3|W|^2$ term.

$$V = e^K \left( K^{a\bar{b}} D_a W \overline{D_{\bar{b}} W} \right) = e^K \|DW\|^2 \geq 0$$

Bound from [Rudelius,'21] not applicable!

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Possible caveats:

- We have not considered Kähler moduli: Stabilized? Also enter? 😕

Analogue potentials can be obtained in **IIA through mirror symmetry**: no violation 😑

# More examples from F-th. + Asympt. Hodge Theory!

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We examine the different possible infinite distance singularity enhancements for $h^{3,1}(Y_4)=2$.

Interesting asymptotic solutions are found! (Taken from [Grimm,Li,Valenzuela,'20])

Type IIB @ LV

| Enhancement | Potential | $\vec{\beta}$ | $\gamma_{\vec{f}}$ | Comments |
|---|---|---|---|---|
| $II_{0,1}\to V_{2,2}$ | $\frac{A_{30}}{su^3}+\frac{A_{32}}{us}+A_{34}\frac{u}{s}+A_{36}\frac{u^3}{s}+A_{52}\frac{s}{u^3}+A_{44}-A_{loc}$ | (3,1) | $\sqrt{\frac{2}{3}}$ (*) | Saturates $c_4^{\text{TCC}}$ After axion stabilization $\gamma_{\vec{f}}\to 2\sqrt{\frac{2}{3}}$ |
| | $\frac{A_{30}}{su^3}+\frac{A_{32}}{us}+A_{34}\frac{u}{s}+A_{36}\frac{u^3}{s}+A_{44}-A_{loc}$ | (1,0) | $\sqrt{2}$ | Saturates $c_4^{\text{strong}}$, stabilized $u_0=\sqrt{\frac{-A_{34}+\sqrt{A_{34}^2+8A_{32}A_{36}}}{4A_{36}}}$, flat $\phi^2$ |
| | $\frac{A_{32}}{su^3}+\frac{A_{32}}{us}+A_{34}\frac{u}{s}+A_{52}\frac{s}{u^3}+A_{44}-A_{loc}$ | (2,1) | $\sqrt{\frac{2}{7}}$ | Violates $c_4^{\text{TCC}}$ |
| | $\frac{A_{30}}{su^3}+\frac{A_{32}}{us}+A_{34}\frac{u}{s}+A_{44}-A_{loc}$ | (1,0) | $\sqrt{2}$ | Saturates $c_4^{\text{strong}}$, stabilized $u_0=\left(3\frac{A_{30}}{A_{34}}\right)^{\frac{1}{4}}$, flat $\phi^2$ |
| $II_{0,1}\to III_{0,0}$ | $\frac{A_{32}}{us}+A_{34}\frac{u}{s}+A_{44}-A_{loc}$ | (1,0) | $\sqrt{2}$ | Saturates $c_4^{\text{strong}}$,stabilized $u_0=\sqrt{\frac{A_{32}}{A_{34}}}$, flat $\phi^2$ |
| $III_{1,1}\to V_{2,2}$ | $\frac{A_{20}}{s^2u^2}+\frac{A_{22}}{s^2}+\frac{A_{42}}{u^2}+A_{24}\frac{u^2}{s^2}+A_{64}\frac{s^2}{u^2}+A_{44}-A_{loc}$ | (1,1) | $\sqrt{2}$ | Saturates $c_d^{\text{strong}}$ |
| | $\frac{A_{20}}{s^2u^2}+\frac{A_{22}}{s^2}+\frac{A_{42}}{u^2}+A_{24}\frac{u^2}{s^2}+A_{44}-A_{loc}$ | (2,1) | $\frac{2}{\sqrt{5}}$ | Violates $c_d^{\text{strong}}$ |
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Generalizable to more moduli (*)

Only Complex Structure moduli taken into account! ⚠️

# Some comments and conclusions!

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- Scalar potentials for CS moduli allow for dS coefficients lower than some proposed bounds!
- We need potentials coming from $\mathcal{E}_{\text{rest}}$:
  - Competing terms allow for lower $\gamma$.
  - Unique trajectory towards infinity independent of starting point!

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- Scalar potentials for CS moduli allow for dS coefficients lower than some proposed bounds!
- We need potentials coming from $\mathcal{E}_{\text{rest}}$:
  - Competing terms allow for lower $\gamma$.
  - Unique trajectory towards infinity independent of starting point!
- What about the Kähler moduli?
  - Should be stabilized “somehow” (DGKT, etc): **Does this affect the shape of our potentials?**
  - Independent runaway makes $\gamma$ too high.
  - Type IIA examples studied (after mirror symmetry) do not violate bounds.

# Relation with other Swampland Conjectures!

- **WGC for membranes** [Lanza,Marchesano,Martucci,Valenzuela,'21]
  Membranes can serve as sources for potential

$$\gamma = \frac{\|\nabla V\|}{V} \overset{?}{\rightarrow} \left(\frac{Q}{\mathcal{T}}\right)^2$$

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- **Swampland Distance Conjecture** [Ooguri,Palti,Shiu,Vafa,'18]: Relation between $\gamma$ and $\alpha$ with $m(\Delta\phi) \sim e^{-\alpha\Delta\phi}$.

Asymptotic runway $V \sim m^{\chi} \implies \gamma = \chi\alpha$ Bound comparison!

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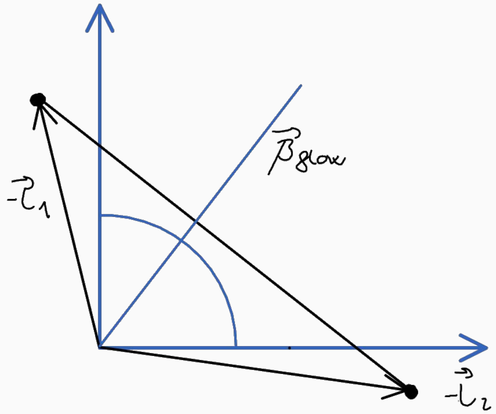
Bound comparison!



- **Convex Hull** [Calderón-Infante,Uranga, Valenzuela,'20]: Same approach!

# Thanks for your attention!

**Paper coming soon!**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